New and Improved Implementation of the First Order Model for Landfill Gas Generation or Collection

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ABSTRACT
The solid waste industry widely uses the US EPA LandGEM model for estimating landfill gas (LFG) generation and collection rates. This model is based on the typical first order decay equation where the rate of decay of a waste mass is dependent on the amount of waste mass.

Currently, LandGEM model uses two parameters, $L_o$ (potential methane generation capacity) and $k$ (methane generation rate), to estimate LFG generation. For a particular waste mass placed in the landfill, the LandGEM model does not allow these parameters to change with time. With advances in landfilling techniques (e.g., bioreactors/leachate recirculation, organics diversion, and stormwater/infiltration management) and changes in the climate that affects precipitation, we have found that in reality $L_o$ and $k$ values are not constant over the life of a landfill. Thus, in order to improve upon and advance the use of LFG emission models as a tool for estimating LFG generation and collection rates, there is a need to develop a model that handles a varying $L_o$ and $k$.

This paper will present a new and improved LFG emissions model that allows $L_o$ and $k$ to be varied at any particular time. It will be compared against the LandGEM model and various other improvements will be discussed, for example: cumulated generation rates will never be greater than $L_o$.

BACKGROUND
LandGEM is widely used in the LFG industry to estimate air pollutant emissions, to size collection systems, and to determine the feasibility of LFG energy projects at specific landfill sites. Details on the LandGEM model could be found in the 1998 (v2.01) and 2005 (v3.02) user manuals referenced herein. LandGEM is relatively easy to use and takes a simple approach to estimating landfill emissions. The model uses only two parameters for estimating the emissions – $L_o$ and $k$. $L_o$ is the potential methane (CH$_4$) generation capacity of the waste mass or the amount of CH$_4$ generated per unit waste mass upon complete decay of the waste mass. $k$ is the waste mass decay rate per unit time (relative to the exponential mass decay equation) or CH$_4$ emission rate variable (relative to CH$_4$ emissions).

One of the significant limitations of LandGEM is that for purposes of modeling, $L_o$ and $k$ are fixed over the entire life of a landfill site. This can be problematic if the waste composition, moisture content, nutrients, or climatic conditions vary over time. Specifically, advanced landfilling techniques such as leachate recirculation and the implementation of bioreactor landfills coupled with increased organics division, in particular an industry trend targeting the recovery and management of food waste for anaerobic digesters and composting, and the increased recycling rates of paper, plastic and metals creates changing conditions that are not satisfactorily addressed with $L_o$ and $k$ values that are fixed over the life of the site. This said, there is a need for a new and improved LFG model that provides flexibility to vary $L_o$ and $k$ values over the life of the site.

LandGEM
Since LandGEM was deployed by the US EPA in the 1990s, it has gone through a few modifications to improve its capabilities and functionality. Throughout these modifications, the basis of LandGEM remains the first-order decay equation Eq. [1].

$$\frac{dM_t}{dt} = -k M_t$$  \hspace{1cm} (1)

where

$M_t$ = mass of waste at time $t$,  
$t$ = time since start of anaerobic decomposition.

This is a first-order differential equation and states that the amount of gas generation is dependent on the amount of mass present at the time. The solution to Eq. [1] is
presented graphically in Figure 1. It is the common exponential function and shows that the waste mass degrades quickly initially, but as time progress this decay rate slows down:

\[ M_t = M_0 e^{-kt} \]  
\[ \text{(2)} \]

where

\[ M_0 \] = mass of waste at time zero.

FIGURE 1. SOLUTION TO THE 1ST ORDER DECAY EQUATION

\[ L_o \] is the potential CH\(_4\) generation capacity of the waste mass. Multiplying Eq. [2] by \( L_o \), waste mass is converted to volumetric CH\(_4\).

\[ S_t = M_0 L_o e^{-kt} \]  
\[ \text{(3)} \]

where

\[ S_t \] = amount of CH\(_4\) that remains to be generated at time \( t \).

And cumulative CH\(_4\) generation is calculated by subtracting \( S_t \) (i.e., Eq. [3]) from \( M_0 L_o \) giving

\[ V_t = M_0 L_o (1 - e^{-kt}) \]  
\[ \text{(4)} \]

where

\[ V_t \] = cumulative CH\(_4\) generated up to time \( t \).

Eqs. [3] and [4] are presented graphically in Figure 2. It shows that the cumulated CH\(_4\) begins at zero and approaches \( M_0 L_o \) exponentially.

LandGEM is derived from Eq. [4]. Refer to Figure 3 which presents cumulative CH\(_4\) as was also presented in Figure 2. In LandGEM v2.01, the volumetric CH\(_4\) generation rate during a certain year \( t \) is calculated as the slope of the curve (or derivative) at the beginning of the year \( t \). These slopes or CH\(_4\) generation rates are summed up for each annual waste mass, eventually arriving at a total CH\(_4\) generation rate or LFG generation rate for the landfill. LandGEM v2.01 mathematical equation is also presented in Figure 3.

Recognizing that the cumulative curve is exponential and that that slopes or CH\(_4\) generation rates decrease during the year, it was recognized that CH\(_4\) rates were being overestimated. As such, in LandGEM v3.02, each year was divided in 10 equal parts and the slopes were calculated at these 10 points and averaged. Refer to Figure 4.

FIGURE 2. CH\(_4\) REMAINING AND CUMULATIVE CH\(_4\)

FIGURE 3. LandGEM V2.01

For waste placement activity spread over a number of years

\[ Q_t = \frac{1}{t} \sum M_i L_o e^{-kt} \]

\[ Q_t = \text{LandGEM ver 2.01 emissions rate} \]
\[ L_o = \text{potential CH}4 \text{ generation capacity} \]
\[ k = \text{CH}4 \text{ generation rate} \]
\[ t = \text{year of calculation} \]
\[ M_i = \text{waste mass placed during year } i \]
\[ t_i = \text{age of the waste mass placed during year } i \]
While this update is an improvement to the initial LandGEM v2.01 model, it is not without inherent inaccuracy. Due to the changing nature of the cumulative CH₄ curve (i.e., its exponential nature), using an arithmetic average based on 10 equal time increments disproportionately weights the average value towards a slightly higher value.

Because of this overestimation, the cumulative CH₄ generation is usually also over projected. Figure 5 shows that LandGEM v2.01 and v3.02 estimates for cumulative CH₄ generation exceed the potential CH₄ generation capacity of the waste mass placed.

Presented mathematically our model is as follows:

\[ Q_n = S_n \left( 1 - e^{-k_n \Delta t_n} \right) / \Delta t_n \]
\[ S_{n+1} = S_n e^{-k_n \Delta t_n} + 2L_{o,n} M_n \]  
(5)

where:
- \( Q_n \) = LFG emissions during the \( n^{th} \) time period;
- \( S_n \) = potential LFG generation capacity at the beginning of the \( n^{th} \) time period, which we refer to as the LFG bank later in the paper;
- \( L_{o,n} \) = potential CH₄ generation capacity of the waste placed during the \( n^{th} \) time period;
- \( k_n \) = CH₄ emissions rate constant during the \( n^{th} \) time period;
- \( M_n \) = amount of waste placed during the \( n^{th} \) time period; and
- \( \Delta t_n \) = duration of the \( n^{th} \) time period.

In this formulation, we assume that LFG consists of 50 percent CH₄ by volume.

We refer to our new model as a marriage between the LandGEM model and the IPCC model (IPCC, 2006) developed for greenhouse gas emissions. We use the concept of \( k \) (relative to CH₄ emissions) and \( L_{o} \) from LandGEM, and we use the IPCC straight forward approach to solving the first order equation.

Figure 6 provides a conceptual diagram for the new model. Waste placed during each period is converted to LFG (by multiplying the waste amount, \( M \), by \( 2L_{o} \), assuming 50 percent CH₄) and “stored” in an LFG bank. Note that this LFG bank is nonliteral; in reality solid waste is stored in the landfill and LFG is released over time as a byproduct of the decomposition process.

LFG is then withdrawn/emitted from the bank at a rate of \( k \) over the period. It’s analogous to monies debited and credited into a bank account earning interest rate \( k \), but in reverse; i.e., the bank account is being depleted at a rate of \( k \). Interestingly enough, the formulas derived herein are very similar to those used by financial institutions.

As the new model is derived from the simplest form of the first-order equation, the mathematics is dramatically simplified. Refer to Eq. [5] and the LandGEM v3.02 equation in Figure 4. The summation that is in LandGEM disappears and \( k \) is only in the exponent. This
allows for varying $k$ and $L_o$ at the beginning of each period/year—a very powerful capability that is not possible in LandGEM. In LandGEM, only a single $k$ and a single $L_o$ could be selected for the entire life of the landfill, or some modelers develop elaborate spreadsheets (e.g., summing separate LandGEM models for different waste types) to implement a variable $L_o$ and $k$.

This ability to vary $k$ and $L_o$ is important for the LFG industry. For example, it is known that $k$ is affected by moisture and nutrients in the landfill and $L_o$ is affected by waste characteristics. Today’s advancing landfilling techniques such as bioreactors, leachate recirculation and organics diversion as well as climate change and infiltration/stormwater management affect moisture levels in landfills and/or waste characteristics which in turn require adjustments in $k$ and $L_o$ values in order to accurately model the decay of the waste. We now have a tool to model LFG emissions in these situations.

FIGURE 6. CONCEPTUAL DIAGRAM FOR THE NEW MODEL

VALIDATION

In order to validate the new model, a US (Virginia) landfill was modeled with LandGEM v3.02 and our new model and the results were compared. $L_o$ and $k$ were held constant in both models.

Figure 7 presents the results. Both models give almost identical results, with a negligible and constant 0.2 percent difference. This difference is a result of the averaging of 10 generation rates during a year described earlier for the LandGEM v3.02 model. It could be shown that this difference is a function of $k$, and the difference increases for sites with higher $k$ values. In this example, $k$ is 0.04.

FIGURE 7. COMPARISON OF LANDGEM V 3.02 AND NEW MODEL

SAMPLE SPREADSHEET AND EXAMPLE MODEL RUNS VARYING $L_o$ AND $k$

Because the new model calculates the addition of waste to the landfill and generation of LFG from the landfill using a basic “debits and credits” approach the model calculation becomes much simpler while yielding the same effective result. LandGEM 3.02 utilizes Microsoft Excel’s array function to perform a summation calculation, or utilizes a long expansive calculation sheet. This new model presents the calculation in a simple and easier to understand format, which allows industry professionals not accustomed to mathematical modeling to more quickly grasp the concepts employed.

Figures 8 and 9 demonstrate the simplicity of the model. In this spreadsheet, no cells are hidden; only the cells shown are utilized in the calculations. Figure 8 illustrates a varying $L_o$ example; and Figure 9 a varying $k$. Note that $L_o$ and $k$ could be change for any period/year.
The paper is summarized below.

1. The new model is an advancement in LFG modeling where $k$ and $L_0$ could be varied during any period.
2. It’s a new tool for estimating LFG emissions from sites considering advanced landfilling techniques such as leachate recirculation, bioreactors, organics diversion, i.e., situations that affect $k$ and $L_0$. It could also be used at sites where the moisture content of the waste is affected by climate change and/or stormwater/infiltration management.
3. The new model is a further simplification of the LandGEM model. For a single $k$ and $L_0$, the results are effectively identical. As it is an exact representation of the first order decay equation, some of the inherent approximations in LandGEM are removed.
4. It's simple.

REFERENCES

